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Long range interactions and the 3D asymptotic fermion spectrum

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Abstract

In this paper, we study the stability of the space of asymptotic fermion states in (2+1)D, when long range interparticle interactions are present. This is done in the framework of bosonization, where the fermion propagator can be represented in terms of a vortex correlator. In particular, we discuss possible instabilities in the large distance behaviour of the induced action for the vortex worldline.

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1. Introduction

In two-dimensional spacetime, bosonization enables the identification of fermions with finite energy solitons of a dual sine-Gordon model [1]. These solitons are topological objects that interpolate between two different vacua $\phi(x \rightarrow -\infty) = 0$ and $\phi(x \rightarrow +\infty) = 2\pi$; for the antisoliton the behaviour is similar, just interchanging the vacua. Many important consequences can be easily derived in the dual scenario. In particular, when a fermion interaction coming from the minimal coupling with a Maxwell term is introduced (Schwinger model) [2], a mass term ϕ^2 is induced in the dual theory. As a consequence, a configuration with nonzero topological charge (it tends to a nonzero value either at $+\infty$ or $-\infty$) cannot have finite energy. In other words, a single soliton, or in fermionic language the single fermion, disappears from the space of asymptotic states. This can be understood as due to the confining nature of the Maxwell interaction in two dimensions, which implies a linear potential between charges.

A natural question that arises is whether a possible destabilization of the fermionic spectrum in higher dimensional systems can be studied in a similar manner. In reference

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[3], fermions in three dimensions have been associated with vortex configurations of the dual bosonized gauge theory. Then, the above-mentioned programme can be rephrased in terms of possible instabilities of the vortex modes as a consequence of interactions.

As we will see, in the three-dimensional case, interactions do not render in general the vortex energy infrared divergent, and it is harder to display scenarios where the destabilization of the vortices is observed. This situation is expected as we know that while in two dimensions the fermion spectrum is easily destabilized by interactions, in three or more dimensions, the quasiparticle picture is quite robust.

Then, we will search for possible destabilizations associated with the localization properties of the dual model. This aim can be understood in the following manner. The association of vortices in the bosonized theory and fermion states has been accomplished by using a path integral definition of the vortex propagator, by creating and annihilating the vortex at the position of an instanton anti-instanton pair in Euclidean spacetime [3]. These Euclidean objects can be seen as a monopole–anti-monopole pair joined by a Dirac string which becomes observable. When the theory presents a well-defined localization length, as in the case of free fermions, the Dirac string can be seen as the vortex worldline and at large distances the induced action is dominated by the length of the string plus Polyakov’s action for spin one half particles. This leads to a pole in the vortex propagator, associated with the single fermion state. Therefore, a possible breakdown of the above-mentioned large distance behaviour, induced by the fermion interactions, could eliminate the single fermion from the asymptotic space of states. We will show that, although the part of the induced action proportional to the string length is robust, there is a set of interactions that could destabilize the spin part.

The paper is organized as follows. In section 2 we describe a general model of fermions coupled to $U(1)$ gauge fields and discuss the dual theory in (1+1)D and (2+1)D, emphasizing the role of topological excitations. In section 3 we analyse possible destabilizations of the fermion propagator due to interactions. Finally, in section 4 we discuss our results.

2. Fermions coupled to $U(1)$ gauge fields

Let us consider massive fermions coupled with a gauge field

$$K_F[\psi] + \int d^v x \left[\mathcal{A} j^F + \frac{1}{4} \mathcal{F}_{\mu\nu} \hat{\mathcal{P}} \mathcal{F}_{\mu\nu} \right], \quad (2.1)$$

where K_F is the action for free fermions, $\mathcal{F}_{\mu\nu}$ is the field strength tensor for a gauge field, and we have included a possible nonlocal operator $\hat{\mathcal{P}}$, associated with a kernel $\mathcal{P}(x - y)$,

$$[\hat{\mathcal{P}}\mathcal{F}](x) = \int d^v y \mathcal{P}(x - y) \mathcal{F}(y).$$

We also define the Fourier transform

$$\tilde{\mathcal{P}}(k) = \int d^v x e^{ikx} \mathcal{P}(x). \quad (2.2)$$

The usual Maxwell term is recovered by taking $\hat{\mathcal{P}} \equiv I$, that is, $\mathcal{P}(x - y) = \delta^{(v)}(x - y)$.

The integration over the gauge fields yields for the total fermion action

$$K_F[\psi] + I[j^F], \quad I[j^F] = \frac{1}{2} \int d^v x j^F \frac{1}{(-\partial^2) \hat{\mathcal{P}}} j^F, \quad (2.3)$$

where v is the spacetime dimension.

2.1. (1+1)D massive fermions

In the simple case of massive fermions in (1+1)D, the model in equation (2.3) can be bosonized according to [1],

$$K_F[\psi] + I[j^F] \leftrightarrow K_B[\phi] + I[\epsilon \partial \phi], \quad (2.4)$$

$$K_B[\phi] = \int d^2x \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \mu \cos \beta \phi \right], \quad I[\epsilon \partial \phi] = \frac{1}{2} \int d^2x \phi \frac{1}{\hat{\mathcal{P}}} \phi, \quad (2.5)$$

where we have used the bosonization rule for the fermion current $j^F = \epsilon \partial \phi$, or in components, $j_\mu^F = \epsilon_{\mu\nu} \partial_\nu \phi$. When the interaction induced by the gauge fields is absent, the model contains the well-known solitons representing the fermions in the bosonized language. These solitons are well-localized finite energy solutions with nonzero topological charge. When $\hat{\mathcal{P}} \equiv I$, the mass term in equation (2.5) implies that configurations with nonzero topological charge cannot have finite energy, as they are different from zero either at spatial $+\infty$ or $-\infty$. In other words, the Maxwell interaction in (1+1)D eliminates the single soliton modes. This can be understood as in this case the fermions experience a linear interaction.

An interesting question arises if we consider a general Lorentz invariant kernel $\hat{\mathcal{P}} = \mathcal{P}(\partial^2)$ in equation (2.5). Is there any situation in which the generally nonlocal ϕ^2 term could allow for finite energy solutions in the infrared regime? For this aim, we can consider a static field

$$\phi(\mathbf{x}) = \int_{-\infty}^{+\infty} d\mathbf{k} \tilde{\phi}(\mathbf{k}) \exp(-i\mathbf{k}\mathbf{x}), \quad (2.6)$$

with nonzero topological charge, thus satisfying

$$\int_{-\infty}^{+\infty} d\mathbf{x} \partial_x \phi(\mathbf{x}) \neq 0, \quad (2.7)$$

this means $\tilde{\phi}(\mathbf{k}) \sim 1/\mathbf{k}$, for $\mathbf{k} \rightarrow 0$. On the other hand, using the Fourier transformed variables, the nonlocal ϕ^2 term reads

$$\int d\mathbf{k} \frac{1}{\hat{\mathcal{P}}(\mathbf{k})} |\tilde{\phi}(\mathbf{k})|^2. \quad (2.8)$$

Therefore, considering the class of operators having

$$\hat{\mathcal{P}}(k) = \lambda/k^\alpha, \quad (2.9)$$

the energy of a nonzero topological configuration will be infrared finite whenever $\alpha > 1$. The static potential $V(\mathbf{x})$ associated with this nonlocal operator,

$$\hat{\mathcal{P}} \partial_{\mathbf{x}}^2 V = \delta(\mathbf{x}), \quad (2.10)$$

corresponds to $V(\mathbf{x}) \propto |\mathbf{x}|^{1-\alpha}$. Then, the above-mentioned energy is infrared finite whenever the nonlocal character of the Maxwell term drives the static potential into a nonconfining one.

2.2. Bosonization and the vortex propagator in (2+1)D

Studying two-dimensional interacting fermionic systems is more difficult than the one-dimensional case, since in (2+1)D only few exact results are available (see for example [4–6]). In this case, it is also well established that the correlation functions of $U(1)$ fermionic currents correspond to correlation functions of topological currents in the dual bosonized theory [7, 8]. This feature has a universal character, generalizing equation (2.4). In other words, we have the following formula [9, 10]:

$$K_F[\psi] + I[j^F] \leftrightarrow K_B[\lambda] + I[\epsilon \partial \lambda], \quad (2.11)$$

where K_B is the corresponding bosonized action and the bosonizing field λ is a scalar field ϕ in (1+1)D, and a gauge field A_μ in (2+1)D. Accordingly, $\varepsilon\partial\lambda$ has to be read as $\varepsilon_{\mu\nu}\partial_\nu\phi$ or $f_\mu = \varepsilon_{\mu\nu\rho}\partial_\nu A_\rho$, respectively.

When parity breaking fermions in (2+1)D are considered, and a large mass expansion is performed, the dominant part of K_B reduces to the Maxwell–Chern–Simons (MCS) model [7, 8],

$$K_B[A] \sim \int d^3x \left(\frac{1}{2m} f_\mu^2 + \frac{i}{2\eta} A_\mu f^\mu \right), \quad (2.12)$$

where m is proportional to the fermion mass and η is the Chern–Simons coefficient in the fermion effective action. Using the universal mapping (2.11), the bosonized form of the total fermion action in equation (2.3), including interactions, turns out to be

$$S[A] = \int d^3x \left(\frac{1}{2} f_\mu \hat{O} f_\mu + \frac{i}{2\eta} A_\mu f^\mu \right), \quad (2.13)$$

$$\hat{O} = \frac{1}{m} + \frac{1}{(-\partial^2)\hat{P}}. \quad (2.14)$$

In a similar manner to the (1+1)D case, where free fermions can be associated with soliton configurations in the dual massive sine-Gordon model, in the case of parity breaking matter in (2+1)D, they can be associated with vortices of the bosonized dual theory.

Following throughout the Hooft procedure [11], the vortex propagator in Euclidean spacetime is obtained by path integrating over configurations where a vortex excitation is created out of the vacuum at a spacetime point x_1 and after an intermediate propagation is annihilated at x_2 . Before x_1 and after x_2 the topological charge vanishes, while it is nonvanishing in between due to the existence of the vortex. Therefore, suitable instanton–anti-instanton singularities have to be introduced at x_1 and x_2 in order to match these inequivalent topological configurations. In the present three-dimensional case these singularities can be seen as a monopole–anti-monopole pair [12, 13] for the dual field strength f_μ , located at x_1 and x_2 , respectively. This pair is associated with a Dirac string γ running from x_1 to x_2 , which becomes observable in the MCS model.

Therefore, the two-point vortex correlation function is defined by

$$\mathcal{G}(x_1 - x_2) = \int d\gamma \int DA e^{-S[A, J]} = \int d\gamma e^{-\Gamma_\gamma}, \quad (2.15)$$

where $S[A, J]$ represents the coupling of the string and the MCS model and Γ_γ is the induced string action obtained by integrating over all gauge configurations in a fixed string background. As discussed in [3], for the coupling

$$S[A, J] = \int d^3x \left(\frac{1}{2m} (f_\mu + J_\mu) \hat{O} (f_\mu + J_\mu) + \frac{i}{2\eta} A_\mu \mathcal{F}^\mu + i\vartheta A_\mu J^\mu \right), \quad (2.16)$$

$$J^\mu(x) = \int_\gamma dy^\mu \delta^{(3)}(x - y), \quad \eta\vartheta^2 = 2\pi, \quad (2.17)$$

the effective action turns out to be

$$\begin{aligned} \Gamma_\gamma = i\pi \int d^3x J_\mu (\varepsilon\partial)_{\mu\nu}^{-1} J_\nu + \frac{1}{2} (1 - \vartheta\eta)^2 \int d^3x J_\mu \frac{\hat{O}}{1 - \eta^2 \hat{O}^2 \partial^2} J_\mu \\ + \frac{i}{2\eta} (1 - \vartheta\eta)^2 \int d^3x J_\mu \frac{\hat{O}^2 \partial^2}{1 - \eta^2 \hat{O}^2 \partial^2} (\varepsilon\partial)_{\mu\nu}^{-1} J_\nu, \end{aligned} \quad (2.18)$$

where $(\varepsilon\partial)_{\mu\nu}^{-1}$ is the Green function for the operator $(\varepsilon\partial)_{\mu\nu}$.

For large mass free fermions, the leading terms of the induced action are [3]

$$\Gamma_\gamma \sim \lambda m L + i\pi S_\gamma, \quad S_\gamma = \int d^3x J_\mu(\varepsilon\partial)_{\mu\nu}^{-1} J_\nu, \quad (2.19)$$

that is, Polyakov’s Bose–Fermi transmutation occurs [14, 15] and the vortex propagator turns out to be that of a spin one-half fermionic excitation,

$$\int d^3p \frac{1}{\sigma^\mu p_\mu + \lambda m} e^{ip(x_1-x_2)}, \quad (2.20)$$

where σ^μ are the Pauli matrices; for more details, see [3] and references therein.

3. Analysis of the stability of vortex modes

Now, when interactions are turned on, we will first search for possible infrared divergences of the vortex energy, similar to those occurring in the single soliton sector in the (1+1)D case. The vortex energy can be read from equation (2.18) as the action per unit length for a straight string running along the Euclidean time direction,

$$E = \frac{1}{2}(1 - \vartheta\eta)^2 \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\tilde{\mathbf{O}}}{1 + \eta^2 \tilde{\mathbf{O}}^2 \mathbf{k}^2}, \quad (3.1)$$

where the quantities in boldface correspond to the two-dimensional projection $k \rightarrow (0, \mathbf{k})$. Let us consider the class of interactions defined by equation (2.9). It is easy to see that now, in the (2+1)D case, there is no α that can render the integral in equation (3.1) infrared divergent. This is clear if \mathcal{P} is such that $\tilde{\mathbf{O}}$ tends to zero or a finite constant, when $|\mathbf{k}| \rightarrow 0$. In the case where $\tilde{\mathbf{O}}$ diverges faster than $|\mathbf{k}|^{-1}$, the infrared behaviour of the integrand is proportional to $(\tilde{\mathbf{O}}\mathbf{k}^2)^{-1}$, which is infrared integrable in two-dimensional momentum space. Finally, when $\tilde{\mathbf{O}}$ diverges like $|\mathbf{k}|^{-1}$ or slower, the integrand is proportional to $\tilde{\mathbf{O}}$, which is also integrable. Then, in (2+1)D there is no energy instability of the vortices. This is understandable as in (2+1)D the quasiparticle picture is quite robust.

The next step is to study a possible destabilization in the localization properties of the theory. Interactions could induce a breakdown of the large distance behaviour in equation (2.19); in this case, the derivation of a propagator which at large distances behaves as the free fermion propagator is not guaranteed. If this happens, instead of a dressed propagator for spin one-half quasiparticles, we could be led to a different phase where the fermion spectrum is destabilized.

For this aim, we will study the effect of interactions, defined by equation (2.1), on the induced string action in equation (2.18), with \hat{O} given in equation (2.14). Note that for $1 < \alpha \leq 2$ in equation (2.9), the associated potential decays faster than the Coulomb $1/R$ potential, in particular, $\alpha = 2$ corresponds to a contact potential. For $\alpha < 1$, the potential decay is slower than the Coulomb potential; $\alpha = 0$ corresponds to a logarithmic potential, $\alpha = -1$ gives a linear confining potential, etc.

Equation (2.18) can also be written in terms of the associated kernels, for instance, the integral in the second term can be written as

$$\frac{1}{2} \int d^3x d^3y J_\mu(x) K(|x - y|^2) J_\mu(y) = \int d\tau d\tau' K(|x(\tau) - x(\tau')|^2), \quad (3.2)$$

$$\begin{aligned} K(|x - y|^2) &= \int d^3k \frac{\tilde{\mathbf{O}}}{1 + \eta^2 \tilde{\mathbf{O}}^2 k^2} e^{-ik(x-y)} \\ &= \frac{4\pi m^2}{r} \int_0^\infty dv \frac{vA(v)}{1 + \eta^2 v^2 A^2(v)} \sin(vr), \end{aligned} \quad (3.3)$$

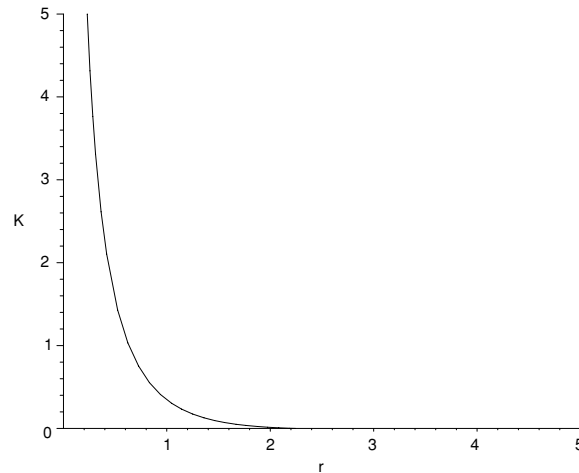


Figure 1. vortex profile with $\alpha = -1$, $\lambda = 2$.

where $x(\tau)$ is a parametrization of γ and we have used that the theory is rotationally invariant as the nonlocal operators depend on ∂^2 . We have also introduced the dimensionless variable, $r = m|x - y|$, and defined

$$A(v) = 1 + \frac{\lambda m^{\alpha-1}}{v^{2-\alpha}}. \quad (3.4)$$

Considering the parametrization $\tau \in [0, L]$, where L is the length of the curve and

$$e_\mu e_\mu = 1, \quad e_\mu = \frac{dx_\mu}{d\tau}, \quad (3.5)$$

and expanding the argument of the kernel in equation (3.2), we obtain

$$|x(\tau) - x(\tau')|^2 = (\tau - \tau')^2 - \frac{2}{4!} \dot{e}_\mu \dot{e}_\mu|_{\tau'} (\tau - \tau')^4 + \dots, \quad (3.6)$$

where we have used that equation (3.5) implies $\dot{e}_\mu e_\mu = 0$ and similar relations.

In general, when the interactions lead to a kernel in equation (3.2) containing a small localization scale δ , in the sense that for $|\tau - \tau'| > \delta$ the kernel is suppressed, we can approximate the argument in equation (3.2) by the first term in equation (3.6), thus obtaining

$$\begin{aligned} & \int_0^L d\tau \int_0^L d\tau' K(|x(\tau) - x(\tau')|^2) \\ & \approx \int_0^L d\tau \int_0^L d\tau' K((\tau - \tau')^2) \\ & \approx \int_0^L d\tau \int_{\tau-\delta}^{\tau+\delta} d\tau' K((\tau - \tau')^2), \end{aligned} \quad (3.7)$$

and changing variables $\tau' \rightarrow \tau' - \tau$,

$$\int_0^L d\tau \int_0^L d\tau' K(|x(\tau) - x(\tau')|^2) \approx \int_0^L d\tau \int_{-\delta}^{\delta} d\tau' K(\tau'^2) = \text{const } L. \quad (3.8)$$

In figure 1 we plot the kernel as a function of r for $\alpha = -1$ and $\lambda = 2$. This kernel turns out to be almost undistinguishable from that obtained in the free fermion model, where $\hat{O} = 1/m$ and the kernel is of Yukawa-type with a localization length of the order of $\delta = 1/m$. We have

also verified that the kernel localization exists over the whole interval of α s, in spite of the fact that the associated potentials are in general long ranged. Therefore, the term proportional to the string length in the large distance induced string action S_γ in equation (2.19) is never destabilized.

Finally, let us look for a possible destabilization of the spin one-half term. First, we note that for free fermions the third term in equation (2.18) is irrelevant at large distances, when compared with the first spin one-half term. However, if the interaction is such that $\tilde{O}^2 k^2$ is nonzero at $k = 0$, the coefficient for the spin one-half action will be changed at large distances and the free fermion propagator could be destabilized. According to equations (2.9) and (2.14), this happens when the associated potential is $R^{-\alpha}$, $\alpha \leq 1$, ranging from a Coulomb up to confining behaviour.

4. Discussions

In this paper we have compared the effect of interactions on the asymptotic fermion spectrum in (1+1)D and (2+1)D. For this aim, we have used the identification of the fermionic excitations with finite energy topological configurations in the dual bosonized theory.

As is well known, in the case of (1+1)D massive fermions, interactions associated with linear potentials destabilize the asymptotic fermionic states in the sense that the energy of a soliton configuration in the dual theory becomes infrared divergent, thus decoupling from the spectrum. This also occurs for any interaction leading to confining interparticle potentials.

On the other hand, we have shown that in (2+1)D the vortex energy is always infrared finite. This is another way of understanding that the free fermionic asymptotic spectrum in higher dimensions is quite robust. However, we have seen that an interesting effect of interactions on the spin degrees of the model takes place. This comes about from the representation of the fermion propagator as a vortex correlator in the dual theory. We have verified that for interactions associated with potentials $R^{-\alpha}$, $\alpha \leq 1$, there is a term in the induced action for the vortex worldline that competes with the spin one-half Polyakov term at large distances. These potentials range from a Coulomb up to confining behaviour ($\alpha = -1$ corresponds to a linear interparticle potential). It would be interesting to investigate the lattice version of the fermionic theory (2.3) to see the cases where this competition is realized as a destabilization of the pole in the fermion propagator.

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